The long-run Phillips curve revisited: Is the NAIRU framework data-consistent?

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Abstract

For the estimation of constant as well as time-varying NAIRUs it is customary to assume –sometimes implicitly– that the long-run Phillips curve is vertical. We point out that the observed data often do not possess the stochastic properties that are needed to impose this restriction, especially when unemployment is non-stationary. Using Germany as a prototypical example, we apply a VAR cointegration analysis and find a negative long-run Phillips curve relation between inflation and unemployment which is robust with respect to variations of the specification. The dynamic interactions indicate that real forces drive the system in the long run, such that the results are compatible with standard economic models.

Keywords: NAIRU, Phillips curve, cointegration, VECM impulse response analysis

JEL classification: C22, E24, E31

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1 Introduction

Mainstream macro models with the classical dichotomy as part of their long-run solution often incorporate a non-accelerating-inflation rate of unemployment (NAIRU).\(^1\) The NAIRU is obviously an important parameter, but attempts to pinpoint its value have often been unsuccessful especially for European data, where large and persistent rises of the unemployment rate suggest that a constant NAIRU is unlikely to exist. Therefore, researchers have tried more sophisticated econometric techniques with time-varying coefficients in order to model this instability in the spirit of e.g. Gordon (1997).\(^2\) However, constant and time-varying NAIRU frameworks all share the feature that the long-run Phillips curve is vertical, because by definition the NAIRU is compatible with any level of inflation. The corresponding empirical assumption is that there is no long-run (i.e. low-frequency) relationship between inflation and unemployment. This assumption is often imposed without testing due to strong theoretical priors about the effect of inflation on unemployment, but it may fail e.g. because the empirical relationship between the two variables is also influenced by the reverse impact of unemployment on inflation. Therefore, in this paper we explicitly test the absence of a long-run connection between unemployment and inflation.

Another motivation for our analysis is that there exists theoretical as well as empirical work which sheds new light on the trade-off between inflation and unemployment. For the USA, Beyer and Farmer (2002) and Ireland (1999) found a positive slope, and Haldane and Quah (1999) make the point that in standard models a “credible optimizing policy maker results in a perverse positively-sloped Phillips curve” (p. 275). Akerlof, Dickens, and Perry (1996, 2000), Karanassou, Sala, and Snower (2005), and Holden (2004) develop or discuss models that explain why there may be a long-run trade-off between output and inflation, especially at low inflation rates. For a panel of EU countries, Karanassou, Sala, and Snower (2003)

\(^{1}\)This implicit definition abstracts from supply shocks which of course may affect inflation. With respect to the acronym it has long been noted that denoting the absence of a change (of inflation) as “non-acceleration” is remarkable for a profession so keen on the proper use of calculus.

\(^{2}\)See for example Apel and Jansson (1999), Laubach (2001), Richardson, Boone, Giorno, Meacci, Rae, and Turner (2000), and Fabiani and Mestre (2001). Some of these studies have been conducted within organizations such as the OECD, the European Central Bank, and a Federal Reserve Bank, so they presumably have some influence on policy makers. For an application to German data see Franz (2001).
present empirical results which imply a long-run inflation-unemployment trade-off. Specifically for the German case, the estimates in Franz (2003) also provide some empirical evidence for a long-run downward-sloping Phillips curve.

Given these different predictions for the long-run inflation-unemployment trade-off, we prefer not to impose strong theoretical restrictions a priori. Instead, we impose tested and thus data-coherent restrictions on the model, using integration and cointegration analysis as well as (structural) vector error correction models, because we hope that these tools may be able to clarify the mysterious relationship between inflation and unemployment (cf. Mankiw, 2001).

Since the empirical analysis of long-run relations requires non-stationary variables, Germany as the prototype of the rising unemployment dilemma in continental Europe is a natural candidate for such an investigation. It is also the largest economy of the euro area with a GDP weight of roughly 30% and thus of particular interest for the monetary policy of the European Central Bank. Indeed, we will show that in Germany the assumption of no long-run relation between inflation and unemployment does not hold. In the sample starting in 1975 with the introduction of the current type of monetary policy there exists a robust empirical long-run relationship between the inflation and unemployment rates. Our estimates indicate the following equilibrium relation: \( \pi_t = 6 - 0.5u_t \), which resembles a “traditional” long-run Phillips curve. However, more detailed analysis shows that this trade-off has been driven by the real side of the economy, not by inflation. Therefore the “traditional” interpretation as a menu for demand-side policy is not supported by our evidence.

The structure of this paper is simple: In the following section 2 we explain our empirical approach and present the test and estimation results. The evidence is discussed in the light of economic theory in section 3. Finally, section 4 offers some conclusions.

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3 It is not the aim of this paper to explain the development of German unemployment, which still remains an unsettled issue as is the case with European unemployment in general.

4 Inflation \( \pi_t \) is measured in annual percentage rates and the unemployment rate \( u_t \) is also in percentages.
2 A cointegration analysis of German inflation and unemployment

2.1 Preliminary considerations

No matter whether a constant or time-varying NAIRU framework is assumed, the steady-state value of unemployment would be given by the NAIRU, whereas for inflation there would be a continuum of endogenous steady-state outcomes along the vertical long-run Phillips curve. Therefore the observability of a vertical (albeit possibly shifting) long-run Phillips curve in actual data requires that the deviations of unemployment from the NAIRU are stationary, whereas the inflation data must be spread out along the continuum of possible long-run values. In other words, the long-run variation of inflation relative to that of unemployment deviations must approach infinity, which makes unit root tests on these variables interesting.

We regard treating inflation and unemployment as potentially $I(1)$ as a testable, pragmatic, and data-coherent modeling device for the non-stationarity in the data, see also Juselius (1999). We are aware that the boundedness of the unemployment rate implies that it cannot remain $I(1)$ forever, and that longer available samples in the future may produce different test results; similar considerations apply to the inflation rate. However, no other accepted model for the highly persistent behavior of unemployment and inflation in continental Europe exists. In fact, even for the less persistent US unemployment data most time-varying NAIRU models describe equilibrium unemployment as $I(1)$ in the state equation, for example see again Gordon (1997). Therefore we are following a widespread practice in the literature.

If inflation and unemployment can statistically be approximated as $I(1)$-processes we have to differentiate between two situations: Either inflation and unemployment do not cointe-

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5See Fair (2000) for an implicit test of this property.
6What if instead inflation is stationary? This would still be compatible with an underlying structural vertical Phillips curve, for instance if policy makers have not tried to influence the real economy through inflation (see Haldane and Quah, 1999, for the formal argument). However, in this latter case we would have to conclude that the observed data cannot be used to apply the empirical NAIRU framework, since an observable vertical long-run curve would not exist.
grate;\textsuperscript{7} then the non-stationary component of unemployment may be removed by fitting a
time-varying NAIRU with the help of purely statistical methods like the Beveridge-Nelson
decomposition, the Hodrick-Prescott or the Kalman filters (see e.g. Hamilton, 1994; such
filtering would leave the equilibrium path unexplained, however). Or there exists coin-
tegration between inflation and unemployment, and there would be no need to filter the
data; indeed, removing the low-frequency component would then almost certainly discard
important information and lead to erroneous conclusions! It is clear that the estimation of
time-varying NAIRUs as is routinely done in the literature would only be warranted if coin-
tegration is not present. However, the needed data properties for this approach are seldom
tested, although such tests would not be difficult. The main contribution of the present paper
is to fill this gap in the literature.

2.2 Data, unit root tests

As the relevant measure for inflation $\pi_t$ we use the first difference of the log of the GDP
deflator,\textsuperscript{8} and for $u_t$ we use the official unemployment rate. We start our analysis after the
strategic monetary policy change by the Bundesbank in 1975 which had an important in-
fluence on inflation control, so we use quarterly observations for the period 1975q2-2002q3.
Using a maximum number of eight lags the effective sample is 1977q2-2002q3. The German
reunification in this statistical data takes place in 1991q1. We consider unadjusted as well as
seasonally adjusted data, where the additive Census X12 procedure is used. Graphs of the
series can be found in figures 1 and 2,\textsuperscript{9} and from the visual inspection of the data we can
draw several conclusions:

- Unemployment has a rising mean over longer horizons. Therefore, it could only be
  stationary around a deterministic trend, if it were stationary at all. The unit root tests

\textsuperscript{7}The concept of cointegration was introduced by Engle and Granger (1987). It means a long-run equilibrium
relation between two or more $I(1)$ variables in the sense that deviations from the equilibrium are stationary.
\textsuperscript{8}There was a change in the national accounts definitions from ESA79 to ESA95 which potentially affects the
inflation rate. (ESA stands for European System of National and Regional Accounts.) The inflation data used
here was recalculated by the German Statistical Office according to the new definition. We also analyzed the CPI
inflation in an earlier version, with qualitatively identical results.
\textsuperscript{9}All empirical results in this paper have been produced with PcGive 10 (Doornik and Hendry, 2001) and
JMulTi (www.jmulti.de).
in table 1 show that trend stationarity is not the case, as the best classification of the series is as $I(1)$ without a deterministic trend, even though we allow for a structural shift. Also note that Papell, Murray, and Ghiblawi (2002) cannot reject the hypothesis of a unit root in the German unemployment rate (among other countries) even after allowing up to five structural shifts.

- The variation of inflation is dominated by seasonal spikes, and it seems that the pattern of the spikes has changed after 1991. Therefore we allow a deterministic structural break of the seasonal pattern for the unadjusted data. Table 2 shows that we are unable to reject the unit root hypothesis for any variant of the series, but note that results are less clearcut than for the unemployment rate. Therefore we will also test the stationarity of inflation in the system context below.

- It is not obvious whether unification had any long-lasting effect such as mean shifts. However, previous applications of appropriate tests in the VAR framework (Johansen, Mosconi, and Nielsen, 2000) revealed that no break is needed in the long-run relations. Therefore in this paper we only include impulse dummies for the transition period, and no step dummy in the long-run relation.

2.3 Cointegration tests

If we choose a vector autoregressive (VAR, in levels) specification as our statistical model, we can test for cointegration between inflation and unemployment with the Johansen rank test, see e.g. Johansen (1995). Formally, the bivariate VAR for the raw data can be described by the following equation:
Here $y_t$ is a vector comprising inflation and unemployment and $m$ is a (two-dimensional) constant term. The innovation vector is given by $v_t$. With respect to the unification shock we use an approach in the spirit of Johansen, Mosconi, and Nielsen (2000) where the transition period after the shock is removed with impulse dummies ($i_{91q1,t}$ describes an impulse dummy for the quarter 1991q1 which has the value 1 in 1991q1 and zeroes elsewhere). For the un-adjusted data we use seasonal dummies including break dummies ($d_{j,t}$ is a centered seasonal dummy which has the value 0.75 in quarter $j$ and $-0.25$ in the other quarters, and has therefore mean zero over a full year; $s_{91q1,t}$ is a step dummy for 1991q1 being zero until 1990q4 and 1 afterwards), and thus we allow for a change of the seasonal pattern in 1991. Bear in mind that the seasonal break dummies do not introduce a changing mean, as each of them has mean zero. For the seasonally adjusted data, the $b_{2,j}$ and $b_{3,j}$ coefficients are simply set to zero. The lag length $K$ is chosen according to various information criteria (allowing a maximum of 8 lags).

Table 3 displays the results of the battery of cointegration rank tests. The central result is of course the clearcut finding of a cointegration relationship between inflation and unemployment and thus there exists a long-run connection between these variables. It does not make a difference whether we use seasonally adjusted or raw data, or whether we use many lags or few.

[Table 3 about here]
2.4 (S)VECM estimates and impulse response analysis

Let us now turn to the estimates of the corresponding vector error correction models (VECMs). Even though cointegration implies that the long-run estimates are super-consistent (i.e. any possible omitted variable bias vanishes asymptotically), for real-world applications it is worthwhile investigating whether the estimates are robust. Therefore we not only consider all specifications of table 3, but we also analyze extensions of the information set. We use the standard “triangle model” approach that includes various measures of supply and demand shocks, see Gordon (1997) and Laubach (2001). We follow those studies by including energy price inflation, imported goods inflation, productivity growth, and exchange rate changes with respect to the US dollar as conditioning variables, see table 4 for the variable definitions. The exogenous variables are seasonally adjusted, wherefore we discard the unadjusted endogenous variables for the analysis with the enlarged information set. For this additional specification with exogenous series all variables are initially allowed to enter both equations with up to three lags. Since the resulting unrestricted system includes quite a few parameters and we lose many degrees of freedom, we apply a general-to-specific selection method, where insignificant lags of endogenous and exogenous variables are subsequently deleted.\footnote{We used the subset model selection procedure in JMulTi (set to the AIC criterion) for that. The cointegrating relation is estimated with a single-equation approach from the inflation equation before the model reduction, using all lags of endogenous and conditioning variables.} The estimation results are reported in table 5.

[Tables 4 and 5 about here.]

The most important lesson from table 5 is that the long-run relation is very robustly estimated. The results may be summarized by writing the equilibrium approximately as:

\[ \pi = 6 - 0.5u \]  

(2)

The next thing to note is that the system estimates in table 5 also provide implicit stationarity tests; this is interesting because the univariate unit root tests for inflation could have been more clearcut. Here the null hypothesis that inflation is \( I(0) \) is equivalent to the state-
ment that the stationary cointegrating relation is already given by $\pi_t + \text{const}$ alone, i.e. that unemployment can be dropped from the cointegration vector. From the reported t-values we see that unemployment is extremely significant, and thus the inflation rate alone is not stationary, confirming the univariate ADF test results.

From table 5 we can also see that inflation bears the error-correction burden. In contrast to that, the unemployment rate reacts to disequilibria with the “wrong” (positive) sign, i.e. deepening the error instead of correcting it. To tackle the interesting question of whether any shocks affect unemployment in the long run it is therefore necessary to perform an impulse response analysis, where the entire system dynamics are taken into account. We compare the VECM including exogenous variables with one that only contains inflation and unemployment, namely the one that also allows three lags (and uses seasonally adjusted data), see the third row of table 5.

The interpretation of impulse response functions usually requires identifying assumptions. However, it turns out that in both analyzed cases the covariance matrix of the (reduced-form) system residuals can be treated as diagonal. This means that the estimated residuals themselves can be interpreted as structural innovations, and the resulting structural vector error correction model (SVECM) is already over-identified. The obtained impulse responses along with the tests of over-identifying restrictions are displayed in figures 3 and 4.

[Figures 3 and 4 about here.]

First of all we see that the two sets of graphs are qualitatively similar. Unemployment shocks cause persistent reactions of both variables of a sign that is consistent with the estimated long-run trade-off, and inflation shocks are mostly insignificant. A difference arises with respect to medium-run reactions to inflation shocks: In the bivariate (i.e. unconditional) system the responses are clearly insignificant, whereas including the mentioned exogenous variables makes the reactions appear borderline significant. It is also interesting that inflation shocks have a “perverse” effect, i.e. an initial increase raises unemployment and thereby leads to a decline of inflation. Therefore the observed negative long-run relation between unemployment and inflation is due to the dominating persistent shocks from unemployment.
Thus there is strong evidence that the source of the non-stationarity of both variables is the real side of the economy, which is represented by the unemployment rate in our system.

3 Discussion

Our findings seem surprising in light of the widespread practice of routinely estimating NAIRUs for monetary policy purposes, because it disputes the data-coherency of the maintained assumptions of many NAIRU analyses. However, our results are nevertheless compatible with standard economic theory. To give a specific and modern example, a negative relation between inflation and unemployment can be derived using a combination of the New-Keynesian Phillips Curve (NPC) theory along with a standard wage-setting framework. A quite general macroeconomic wage curve formulation is provided in Blanchard and Katz (1999), stating the following relation between real wages $w_t$, labor productivity $q_t$, and unemployment $u_t$; for our purposes it is sufficient to subsume all growth terms (differences) and constants in the term $\epsilon^*_t$:

$$w_t - q_t = -\beta (1 - \mu \lambda)^{-1} u_t + \epsilon^*_t \quad \beta > 0; \ 0 \leq \mu, \lambda \leq 1 \quad (3)$$

To obtain a balanced equation with a stationary error term $\epsilon^*_t$ it is either necessary that unemployment $u_t$ and the log labor share $w_t - q_t$ are stationary, or that their long-run developments are related, e.g. in countries with rising unemployment. A declining labor share has indeed been observed in such countries (Blanchard and Wolfers, 2000). And following Galí and Gertler (1999), who suggested to use the labor share as a measure of real marginal costs, the benchmark micro-founded NPC relation is the following:

$$\pi_t = \gamma \sum_{k=0}^{\infty} \rho^k E_t (w - q)_{t+k} \quad \gamma, \rho > 0 \quad (4)$$

This equation states a positive co-movement between the labor share and inflation. Altogether, economic theory predicts a negative relation between unemployment and the labor
share, and a positive relation between the labor share and inflation. This logically implies an observed negative relation between unemployment and inflation. This NPC-based explanation is clearly uni-directional, where the real variables determine inflation. Our impulse response analysis supported this interpretation, e.g. because inflation shocks turned out to be transitory, and if anything they induce unemployment to move in the same direction as inflation. Our findings therefore imply that recent theoretical explanations and interpretations of negative long-run Phillips curves where the shocks stem from monetary policy (Karanassou, Sala, and Snower, 2005; Holden, 2004; Akerlof, Dickens, and Perry, 2000, 1996) do not seem to apply to the German data.

All in all, the rise of German unemployment plays the role of the exogenous factor that is unexplained by our empirical model. Once again it turns out that the Phillips curve framework is well suited as a model for inflation, but not to explain or predict unemployment developments.

4 Conclusions

In this paper we have criticized the prevailing practice of using the NAIRU framework without testing the data compatibility of its maintained assumptions, especially when unemployment or inflation display non-stationary behavior. We argued that in many cases allowing for a time-varying NAIRU does not solve the fundamental problem of discarding information on the potential low-frequency connection between inflation and unemployment.

As the prototype of an economy with high and persistent unemployment, Germany turned out to be a relevant example: Apart from some modeling details, the empirical analysis was quite straightforward and therefore its results can be quickly summarized: The German inflation and unemployment rates in our analyzed sample 1977-2002 are best described as being \( I(1) \) and cointegrated in an inverse relationship, approximately given by \( \pi = 6 - 0.5u \).

The tests and estimates are robust with respect to the inclusion of common conditioning variables like energy and imported inflation, or productivity and exchange rate shocks. However, the persistent shocks from unemployment were the driving source of the observed
developments, and therefore there is no evidence that falling inflation caused rising unemployment. Rather, the reverse direction seemed to be at work.

The implications for the application of NAIRU theories are the following. First, an empirically identifiable constant NAIRU often does not exist. Second, if cointegration between inflation and unemployment is found, it describes an endogenously time-varying steady state in a straightforward manner. Any researcher who wishes to substitute this result by an arbitrarily filtered time-varying NAIRU component may throw away information in the data. In general, our results indicate that forcing the constant or time-varying NAIRU assumption on the data without testing the necessary fundamental time series properties is questionable.

Despite all these reasons, the empirical specification of a non-vertical long-run Phillips curve is almost a taboo for researchers. However, this attitude has ignored the insight that “...to explain the movement of unemployment alongside inflation, one needs to relate unemployment persistence to nominal magnitudes, including inflation persistence.” (Lindbeck and Snower, 1999, p. 87) With the present paper we have tried to take a modest step in this direction.
References


### A Tables

#### Table 1: Unemployment unit root tests

<table>
<thead>
<tr>
<th></th>
<th>seasonally adjusted</th>
<th>unadjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho - 1$</td>
<td>-0.028 -0.016</td>
<td>-0.039 -0.030</td>
</tr>
<tr>
<td>t-stat</td>
<td>-2.29 -2.02</td>
<td>-2.08 -2.64</td>
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<tr>
<td>10% crit. value</td>
<td>-3.50 -3.05</td>
<td>-3.50 -3.05</td>
</tr>
<tr>
<td>lags of diff.</td>
<td>1</td>
<td>1, 4, 5</td>
</tr>
<tr>
<td>Deterministics:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>trend</td>
<td>yes no</td>
<td>yes no</td>
</tr>
<tr>
<td>centered seasonals</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>step dummy $s_{91q1,t}$</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>lags of impulse $i_{91q1,t}$</td>
<td>0, 1</td>
<td>0, 1, 4, 5</td>
</tr>
<tr>
<td>Regression statistics:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sample</td>
<td>75q3-02q3</td>
<td>76q3-02q3</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.62 .62</td>
<td>.93 .93</td>
</tr>
<tr>
<td>no AR (lags 1-8)</td>
<td>.73 .53</td>
<td>.53 .56</td>
</tr>
<tr>
<td>no ARCH (lags 1-4)</td>
<td>.35 .32</td>
<td>.93 .95</td>
</tr>
<tr>
<td>normality</td>
<td>.95 .94</td>
<td>.93 .95</td>
</tr>
</tbody>
</table>

**Notes:** The ADF test equation is $\Delta x_t = (\rho - 1)x_{t-1} + \text{other terms}$; a constant is always included. Diagnostic tests results (last three rows) are given as p-values. The step dummy $s_{91q1,t}$ (for German unification) is zero until 1990q4 and 1 afterwards, and the impulse $i_{91q1,t}$ is its difference.
Table 2: Inflation (GDP deflator growth) unit root tests

<table>
<thead>
<tr>
<th></th>
<th>seasonally adjusted</th>
<th>unadjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho - 1$</td>
<td>$-0.35$</td>
<td>$-0.24$</td>
</tr>
<tr>
<td>t-stat</td>
<td>$-2.71$</td>
<td>$-2.42$</td>
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<tr>
<td>10% crit. value</td>
<td>$-3.05$</td>
<td>$-2.58$</td>
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<tr>
<td>lags of diff.</td>
<td>$1, 2, 3, 4, 6$</td>
<td>$1, 2, 3$</td>
</tr>
</tbody>
</table>

Deterministics:
- centered seasonals (w/ break)
- step dummy $s_{91q1,t}$
- lags of impulse $i_{91q1,t}$

Regression statistics:
- sample
- $R^2$
- no AR (lags 1-8)
- no ARCH (lags 1-4)
- normality

<table>
<thead>
<tr>
<th></th>
<th>76q4-02q3</th>
<th>76q1-02q3</th>
<th>76q1-02q3</th>
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<tbody>
<tr>
<td>$R^2$</td>
<td>.65</td>
<td>.64</td>
<td>.98</td>
</tr>
<tr>
<td>no AR (lags 1-8)</td>
<td>.45</td>
<td>.38</td>
<td>.48</td>
</tr>
<tr>
<td>no ARCH (lags 1-4)</td>
<td>.60</td>
<td>.27</td>
<td>.15</td>
</tr>
<tr>
<td>normality</td>
<td>.59</td>
<td>.40</td>
<td>.85</td>
</tr>
</tbody>
</table>

Notes: The ADF test equation is $\Delta x_t = (\rho - 1)x_{t-1} + \text{other terms}; a constant is always included. Diagnostic tests results (last three rows) are given as p-values. The step dummy $s_{91q1,t}$ (for German unification) is zero until 1990q4 and 1 afterwards, and the impulse $i_{91q1,t}$ is its difference.

Table 3: VAR cointegration test results (Johansen procedure)

<table>
<thead>
<tr>
<th>seasonal adj.</th>
<th>lags</th>
<th>$H_0: r \leq$</th>
<th>trace statistic</th>
<th>eigenvalue</th>
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<tr>
<td>dummies</td>
<td>6 (A, HQ)</td>
<td>0</td>
<td>31.7**</td>
<td>0.24</td>
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<tr>
<td></td>
<td></td>
<td>1</td>
<td>3.2</td>
<td>0.03</td>
</tr>
<tr>
<td>dummies</td>
<td>2 (S)</td>
<td>0</td>
<td>51.4**</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2.6</td>
<td>0.03</td>
</tr>
<tr>
<td>X12</td>
<td>3 (A)</td>
<td>0</td>
<td>28.1**</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>3.0</td>
<td>0.03</td>
</tr>
<tr>
<td>X12</td>
<td>2 (HQ, S)</td>
<td>0</td>
<td>35.3**</td>
<td>0.28</td>
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<tr>
<td></td>
<td></td>
<td>1</td>
<td>2.4</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Notes: Sample 1977q2-2002q3; the VAR is the one described in equation (1) where the constant has been restricted to the cointegration space. The lag length $K$ is chosen according to the information criteria Akaike (A), Hannan-Quinn (HQ), and Schwarz (S). Critical values for the cointegration rank tests (trace stats.) with restricted constant are 19.96 (5%) and 24.60 (1%) for $H_0: r = 0$. Significance indicated by ** (1%), * (5%).
Table 4: Conditioning variables

<table>
<thead>
<tr>
<th>variable name</th>
<th>OECD database denomination</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy price inflation</td>
<td>DEW\DEU CPI Energy\Index publication base /Consumer Price Index\OECD Groups\Energy (Fuel, electricity &amp; gasoline)\Total /2000Y /Cnt: Germany</td>
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<tr>
<td>imported goods inflation</td>
<td>Import Prices /Index Number /Base year: 2000 /averages</td>
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<tr>
<td>productivity growth</td>
<td>Labour Productivity in the business sector Germany (GDP per hour worked /OECD ECONOMIC OUTLOOK)</td>
</tr>
<tr>
<td>exchange rate changes</td>
<td>Market Rate /Euros US dollar per .. /averages Euro Area /Source: IMF; Market Rate /Natl Currency, per US dollar /averages Germany</td>
</tr>
</tbody>
</table>

Notes: First difference of the logarithm is used. The $/euro and euro/DM rates were combined with the fixed euro/DM conversion rate. All conditioning variables are clearly stationary –I(0)– according to standard unit root tests.

Table 5: System (VECM) estimation of long-run parameters

<table>
<thead>
<tr>
<th>setup</th>
<th>lags</th>
<th>ec-term</th>
<th>loading on $\Delta \pi_t$</th>
<th>loading on $\Delta u_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>unadjusted data, bivariate</td>
<td>6</td>
<td>$\pi + 0.50u - 6.0$ (8.0) $(-12.4)$</td>
<td>$-1.0^{**}$ (4.4)</td>
<td>0.042 (1.6)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$\pi + 0.50u - 6.1$ (6.4) $(-10.1)$</td>
<td>$-1.1^{**}$ (7.1)</td>
<td>0.024 (1.4)</td>
</tr>
<tr>
<td>seasonally adj., bivariate</td>
<td>3</td>
<td>$\pi + 0.52u - 6.1$ (6.2) $(-9.5)$</td>
<td>$-0.74^{**}$ (4.9)</td>
<td>0.016 (1.1)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$\pi + 0.54u - 6.4$ (6.6) $(-10.1)$</td>
<td>$-0.81^{**}$ (6.1)</td>
<td>-0.00027 (-0.0)</td>
</tr>
<tr>
<td>with conditioning variables (seasonally adj.)</td>
<td>(see note)</td>
<td>$\pi + 0.54u - 6.1$ (6.2) $(-8.1)$</td>
<td>$-1.12^{**}$ (13.9)</td>
<td>0.037** (2.8)</td>
</tr>
</tbody>
</table>

Notes: t-values in parentheses. Endogenous variables are GDP deflator inflation ($\pi$) and the unemployment rate ($u$), and thus all systems consist of two equations. For unadjusted data centered seasonal dummies with break in 1991 are used. For the lag choice in the bivariate VECMs see table 3. For the system with conditioning variables (energy price inflation, imported goods inflation, labor productivity growth, and exchange rate changes with respect to the US dollar, see table 4 for definitions; all seasonally adjusted) a maximum lag of 3 is allowed for all variables and a model reduction search is conducted (see text); however, exchange rate shocks are dropped by the reduction search. The cointegrating equation for the conditional model is estimated by a nonlinear least-squares single-equation technique from the inflation equation prior to the model reduction (the loading factors are estimated in the system, however). The error correction terms and loadings are defined as in the following example which refers to the first row:

$$
\Delta \pi_t = -1.0(\pi_{t-1} + 0.50u_{t-1} - 6.0) + \text{lagged diff's. and dummies}
$$

$$
\Delta u_t = 0.042(\pi_{t-1} + 0.50u_{t-1} - 6.0) + \text{lagged diff's. and dummies}
$$
B Figures

Figure 1: The unemployment data

Notes: Source DIW (German Institute for Economic Research). Until 1990q4 West Germany, afterwards unified Germany.

Figure 2: The inflation data

Notes: Source DIW (German Institute for Economic Research). Until 1990q4 West Germany, afterwards unified Germany; ESA95 definition. Data are given in approximate annual rates (four times the quarterly rates).
Notes: Estimated impulse responses from the VECM with exogenous variables (see text). The reduced-form residual correlation is not significant (tested as $\chi^2(1) = 1.20, \ p = 0.27$), thus the covariance matrix is restricted to be diagonal. This automatically (over-) identifies the orthogonal structural innovations, and the variable ordering is irrelevant. Confidence intervals (95%) are based on Hall’s bootstrap procedure as proposed by Benkwitz, Lütkepohl, and Wolters (2001) because this procedure has a built-in bias correction in contrast to standard bootstrap confidence intervals.
Figure 4: Impulse responses of the unconditional system

Notes: Estimated impulse responses from the bivariate VECM (GDP deflator growth and unemployment). The reduced-form residual correlation is not significant (tested as $\chi^2(1) = 0.13$, $p = 0.72$), thus the covariance matrix is restricted to be diagonal. This automatically (over-) identifies the orthogonal structural innovations, and the variable ordering is irrelevant. Confidence intervals (95%) are based on Hall’s bootstrap procedure as proposed by Benkwitz, Lütkepohl, and Wolters (2001) because this procedure has a built-in bias correction in contrast to standard bootstrap confidence intervals.