

Pensions and insider-outsider unemployment

by
SVEN SCHREIBER*

If workers gain an insider position through past activity, young workers will bear the resulting outsider unemployment burden. In a world where productivity of employed workers rises because of learning-by-doing, and where labor demand is sufficiently elastic, preventing this unemployment (by lowering wages) leads to a higher income tax base in the future. Thus the institution of certain intergenerational transfer schemes provides an incentive for insiders to lower wages. In a stylized overlapping generations model I show that this effect partially or fully abolishes unemployment in the steady state equilibria.

(JEL: H55, J51, J64)

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1 Introduction

Many industrial countries have unfunded pension systems, and an unfunded pay-as-you-go pillar is likely to remain an important component of the future pension mix. Economists have often pointed to the lower rates of return of unfunded systems as a potential cause of (more) unemployment, because lower returns induce higher pension contributions and thus increase labor costs. Given that mass unemployment still remains a problem in many industrial countries, this reasoning is an important insight and suggests that exclusive reliance on unfunded pensions is not optimal. Nevertheless, real-world labor markets are complex, and there may be additional interactions between labor-market institutions and pay-as-you-go pension systems.

This paper explores the interaction between an unfunded pension system and the particular type of unemployment caused by the wage-setting power of insiders. The insider-outsider theory rests on the idea that in the presence of turnover costs tenured workers are able to demand wages above the market-clearing level, and this idea is an established device to explain at least some part of unemployment in industrial countries, see LINDBECK AND SNOWER [1986]. I follow e.g. PISARIDES [1989] by viewing this as a generational conflict because insiders are older than outsiders, which simply happens because the old generation entered the labor market earlier. In a dynamic overlapping generations (OLG) model I find that a pension system may alleviate the outsider unemployment burden. The underlying idea is quite simple and intuitive: If insiders receive a reward from outsiders, they will have an incentive to “let outsiders in” by lowering wages. Certain intergenerational transfer schemes including a pay-as-you-go pension system may provide such a reward from (former) outsiders to (retired) insiders. Insiders thus forgo some of their instantaneous rents in exchange for future pension income, and the result is that insiders effectively invest in the human capital of the young outsider generation by letting them work. However, to be a sufficiently rewarding “investment project” high employment now must also raise the social security tax base, wherefore I also assume training-on-the-job (or learning-by-doing) effects that increase workers’ incomes over time. Another intuitive requirement is that lowering wages must lead to high enough additional employment, i.e. labor demand must be sufficiently elastic.

Some interactions between social security transfers and human capital issues have already been addressed in earlier literature. MERTON [1983] gives an efficiency argument for social security systems based on the non-tradability of human capital in a risky environment. KONRAD [1995] models the investment in the young generation as the transfer of physical capital and the direct formation of human capital. The contribution on intra-family transfers by CREMER, KESSLER, AND PESTIEAU [1992] is roughly comparable with respect to the schooling as-

pect, but (apart from other features) there the pension scheme does not provide a strategic incentive like in the present paper because of free-rider effects; instead the pension system's influence is indirect via a shifted income position in another strategic game about bequests. Finally, two contributions are worth mentioning that show how pensions can have beneficial effects in an endogenous growth setting. KEMNITZ AND WIGGER [2000] treat human capital as a public good; for its accumulation it is important that pension transfers are related to one's own earnings history, in contrast to the present paper. In WIGGER [2001] intergenerational transfers yield a Pareto improvement in combination with an investment subsidy which could also relate to human instead of physical capital.

While the increase of human capital is also necessary in my model, the treatment of unemployment complements the existing literature. The explicit capital transfer or provision of education is replaced by a learning-by-doing mechanism through which higher employment leads to higher human capital levels. Subsidies of any kind are not needed here. In reality, of course, several strategic (as well as non-strategic) channels could coexist, for instance if formal education, physical capital, and on-the-job training are (partly) complements.

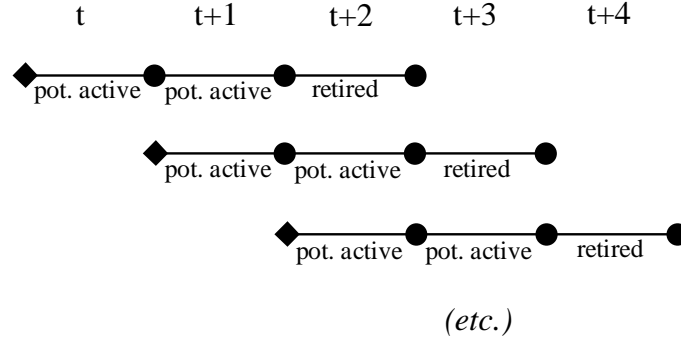
The remainder of this paper is structured as follows: Section 2 describes the assumptions of the model. In section 3 the model is viewed as a dynamic game and conditions for several interesting Nash equilibria are derived, most notably the case of full employment. Then a stronger solution concept without game theory and the conditions for these equilibrium types are analyzed in section 4, and finally section 5 provides concluding comments. All proofs of the results are given in the appendix.

2 The setup

The economy consists of an infinite sequence of overlapping generations that live for three periods. In each period a new generation of individuals is born and immediately enters the labor market. Its members attempt to supply labor for two periods, followed by one period of retirement. For simplicity, there is no population growth, so the cohort size is constant and is normalized to 1. In each period there are three generations alive; young workers (outsiders), old workers (insiders or outsiders, see below), and retirees. A sketch of the structure of this OLG economy is given in figure 1.

Effective labor units are homogeneous, where the effective labor supply of each individual is given by l or h , with $0 < l < h$. I assume learning by doing, so a newly employed worker can initially only supply l , whereas after one period of experience her potential rises to h for the remaining active life span. In the rest of this paper, h is normalized, $h = 1$. The learning-by-doing aspect explains the

Figure 1: The OLG structure



Notes: *Individuals enter the labor market immediately after birth, and they are potentially active (“pot. active”) for two periods, after which they retire.*

necessity of a three-period OLG model with two (potentially) active periods.

Output is produced by a representative firm according to a neoclassical production function with decreasing returns to effective labor inputs, $f[A_t]$, $f' > 0$, $f'' < 0$, where A_t is effective employment in period t . (Note that I use square brackets for the argument of this function throughout this paper.) This formulation embodies some simplifications which were adopted to expose the core of the argument: First, f is constant over time, which means that technological progress is assumed away. Second, effective labor is the only modeled input, such that capital accumulation does not happen in the model. These assumptions create a stationary labor market environment in the sense that competitive wages would not tend to change. This feature is an abstraction from reality which serves to focus on the effect modeled in this paper.

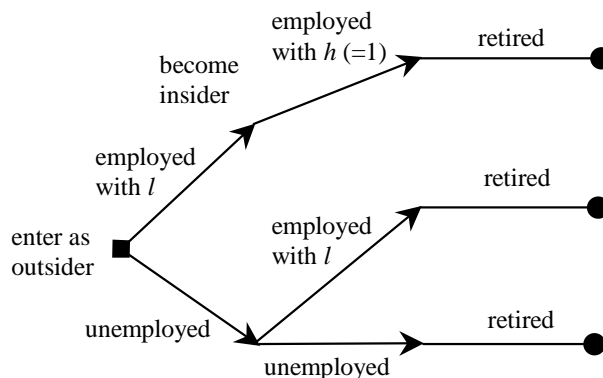
Effective employment consists of both types of workers, whose labor supply is identified by subscripts “low” and “high”; superscripts help to identify which generation supplies the respective amount of labor. By construction of the model there cannot exist young high-productivity workers, such that effective employment becomes:

$$(1) \quad A_t = l (A_{low,t}^{young} + A_{low,t}^{old}) + A_{high,t}^{old}$$

For simplicity, it is assumed that the centralized insider union is in a monopoly position and sets wages accordingly.¹ If there are no insiders in the economy, the labor market outcome is competitive with wages and employment at the level compatible with full employment. Note that it is assumed that there is no wage

¹Bargaining models would complicate the analysis, but would yield little additional insight.

Figure 2: Possible CVs of an individual



discrimination between (employed) effective labor units. Employment levels are set by the (otherwise passive) firm and are therefore determined by the “right-to-manage” condition, simply equating marginal productivity to marginal costs. This merely means that the firm implements the value of its labor demand function given the wage rate W_t (for effective labor) set by the union or determined by competition:

$$(2) \quad f'[A_t] = W_t$$

If wages are set by an insider union at the beginning of a period, I assume that jobs are allocated serving the tenured workers (new insiders) first, which can be motivated by firing costs. However, some insiders may be fired if the wage level exceeds the productivity of the marginal insider. As I also assume insider-biased “solidarity” (or alternatively high risk aversion among insiders), the union will set wages such that all insiders remain employed. Of course, in the absence of strategic considerations this means that the insider union will set wages at the productivity level of the marginal insider, which will cause all outsiders to remain or become unemployed. With this assumption, the event tree describing a worker’s possible *curriculum vitae* looks as shown in figure 2. The formalization of this restriction is given by:

$$(3) \quad A_{high,t}^{old} = A_{low,t-1}^{young}$$

It is useful to introduce the outsider employment quota $q_t \in [0, 1]$ as the central variable describing the evolution of the model. It is assumed that firms do not discriminate between young and old outsiders, randomly drawing the needed workers from the outsider supply pool. With the employment quota the various employed labor quantities can be described as follows; note that these are not

effective units. First, the amount of employed young outsiders in t , who by definition have low productivity, is given by (recall that the cohort size is unity):

$$(4) \quad A_{low,t}^{young} = q_t$$

Because of (3) this is also the amount of insiders in $t + 1$, who are high productivity workers. Next, all workers who were unemployed in their first active period remain outsiders ($1 - q_{t-1}$). The amount of employed old outsiders in t is therefore:

$$(5) \quad A_{low,t}^{old} = q_t(1 - q_{t-1})$$

Plugging these substitutions into the right-to-manage condition (2) and using (1) yields:

$$(6) \quad W_t = f'[lq_t(2 - q_{t-1}) + q_{t-1}]$$

Finally, consider a pension system with a constant contribution rate b . The budget constraint of the pension system then reads as follows:

$$(7) \quad bW_t A_t = O_t,$$

where total pension outlays O_t must be covered by the share (b) of aggregate labor income in the period. Assuming for simplicity equal distribution of the payments among the retired generation (with size normalized to one) O_t is also the relevant pension benefit.²

As the stress in this paper is on purely egoistic motives, without any intergenerational altruism or solidarity, the utility of insiders in period t can be indexed by the following expression:

$$(8) \quad V_t = (1 - b)W_t + \frac{1}{1 + r}bW_{t+1}A_{t+1}$$

The first term is simply labor income net of social security taxes for an individual insider's effective labor supply h ($= 1$). The second component is the transfer received from the pension system, which is the equally distributed (among the retired generation, with size normalized to one) discounted ($1/(1 + r)$) share (b)

²This obviously entails some intra-generational redistribution from formerly well-paid insiders to former low-income outsiders. In reality many pension systems indeed contain intra-generational redistributive elements, although the complete benefit equalization that I have imposed here is clearly counter-factual. Note that insiders would have an additional incentive to increase outsider employment –thereby enlarging the future tax base, at the initial cost of lower wages– if the pension system did not redistribute (parts of) the resulting increase of insiders' pensions among former outsiders.

of the future aggregate labor income, where r is the exogenous time discount rate. Applying the above equalities (1), (3), (4), (5), and (6), the problem of the insiders is given by:

$$(9) \quad \max_{q_t \in [0,1]} V_t = (1 - b)f'[lq_t(2 - q_{t-1}) + q_{t-1}] + \frac{b}{1 + r} f'lq_{t+1}(2 - q_t) + q_t$$

The relevant strategic decision is made by the insiders who set wages taking into account the effect of this decision on the labor market equilibrium and future consequences, especially for productivity levels of younger workers. From (9) it is clear that the decision of the insiders in general also depends on the actions of next period's insiders who set q_{t+1} , which in turn also depends on the current period's action q_t . This interdependence can be tackled in two different ways: First the model can be viewed as an infinite game in extensive form and one can look for (some of) its Nash equilibria. In the next section I therefore provide a game theoretic description of the class of games corresponding to the model setup, and I will derive conditions for the most interesting equilibria. Secondly, one can look for conditions which ensure that the entire path of the economy is predictable by forward induction given an initial situation q_0 . These conditions must necessarily be stronger than the ones for Nash equilibria, because the future must be effectively irrelevant in order to infer the choice of insiders in t only from the past, i.e. the conditions must hold for all possible values of $q_{t+1} \in [0, 1]$. For this forward induction it is an important feature of the model that only the outcome in $t - 1$ matters for insiders in t , not the entire past. The conditions for forward induction equilibria will be discussed in section 4.

Before proceeding to find equilibria of the model, let me clarify the assumption of a fixed contribution rate. At first glance this assumption seems unrealistic, because in most unfunded pension systems the payments to retirees are set proportional to some index of the economy's average wage income ("defined benefit"), which endogenously determines the contribution rate. However, the contribution rates in many mature systems have hit the upper bound of economic and political sustainability, and most recent pension reforms have intentionally broken the link between pension benefits and labor income to keep the contribution rate from rising any further, as can be observed for example in Germany. Therefore it is actually more realistic to approximate future pension systems in those countries by assuming a constant rate b rather than a constant defined-benefit ratio. Furthermore, note that in this model the average net wage income of both active generations in period t is given by $(1 - b)0.5W_tA_t$. Relating the average (per retiree) pension benefits bW_tA_t to the average income yields a replacement ratio of $c = 2b/(1 - b)$. This shows that the replacement rate is a function of the con-

tribution rate alone, and it makes no difference whether we fix the former or the latter.³

3 The game played by insider unions

3.1 The formal description

Let us consider the class of games G arising from the model setup of the previous section, with elements $g(f; b, r, l)$ that are characterized by the production function f and the parameters b, r, l . The game theoretic language I will use here is adapted from OSBORNE AND RUBINSTEIN [1994], but I have avoided some notational technicalities that are only needed for more complicated setups. The infinite set of the non-negative integers represents the relevant time periods t of the economy, where $t \in N$, $N = \{1, 2, 3, \dots\}$. I assume an initial condition of $q_0 = 1$, reflecting clearing markets. Obviously, this is the outcome without an insider union and can thus be naturally interpreted as the situation that a newly founded insider union faces. It will also be shown that without a pension system it happens every other period. A *history* h_t for period t is given by realized employment quota developments in the economy so far: $h_t = \{q_j\}_{j=1}^{t-1}$. The set H_t contains all possible histories of the game up to period t .⁴ Note that $h_1 = H_1 = \emptyset$.

Next, the set of the insider unions of the different periods as the *players* in this game is also given as N , and an individual player is denoted as $i \in N$. The *player function* $P(h_t)$ that tells us which player gets to play after a certain history is also straightforward here, because it only depends on the period, not on the actual history: $P(h_t) = t$ for all finite t . The *action* of each insider union $i = t \in N$ is of course the chosen employment quota q_t , and their preferences $V_{i=t} = V_t(q_{t-1}, q_{t+1}; q_t)$ over the set of possible *outcomes* were already given in (9). Finally, the precise definition of strategies may be worth highlighting.

DEFINITION 1 *A feasible strategy s_i of each insider union $i = t$ specifies possible choices of employment quotas q_t for all possible histories $h_t \in H_t$. A specific strategy profile s aggregates a certain combination of individual strategies and for all players $i \in N$ this can be decomposed into the components $s_{<i}$ (strategies of all other players $j \in N$ satisfying $j < i$), $s_{>i}$ (the same for $j > i$) and s_i (own strategy). Finally I define $s_{-i} \equiv (s_{<i}, s_{>i})$.*

³This equivalence result of course applies only to the present stylized model. In the real world with demographic uncertainty, elastic labor supply, and pension adjustment rules with time lags, the relationship between the contribution rate and the replacement rate becomes more complicated.

⁴Its closed-form description is complicated by the fact that $q_{t-1} = 0$ leads to the absence of insiders in t and thus automatically to $q_t = 1$. Therefore, using M for the set of all histories with patterns of the form $\{\dots, 0, q \neq 1, \dots\}$, and using \times for a Cartesian product, $H_t = \times_{j=1}^{t-1} [0, 1] \setminus M$ holds. H in turn collects all possible histories and is given by the union $H = \cup_{t \in N} H_t$.

Note that for the strategies it does not matter whether the histories will be an equilibrium outcome or not. Since a strategy profile determines the employment quota path outcome, the utility of players can also be written as $V_i = V_i(s) = V_i(s_{-i}; s_i)$. As is usual in extensive games with perfect information I will only analyze pure strategies. The following definition of a Nash equilibrium is merely an application of the standard concept to the specific setup of the present model.

DEFINITION 2 *A Nash equilibrium of the described economy $g(f; b, r, l)$ is any strategy profile s^* that produces an outcome of the employment quota path $\{q_t\}_{t \in N}$ such that for all t ,*

$$(10) \quad V_{i=t}(s_{-i}^*, s_i^*) \geq V_{i=t}(s_{-i}^*, s_i)$$

must hold for all possible own strategies s_i , given the other strategies s_{-i}^ .*

3.2 The Nash equilibrium without a pension system

Now that the description of the dynamic game is complete and before I address some of the possible equilibria for the general case $b > 0$, it is useful to consider the benchmark case without a pension system:

Lemma 1 *If there is no pension system, i.e. $b = 0$, then all equilibria are given by strategy profiles that oblige all insider unions to choose $q_t = 0$ for a history characterized by $q_{t-1} = 1$. The equilibrium outcome determined by these strategy profiles is unique and is given by the infinite cycle $\{q_t\}_{t \in N} = (0, 1, 0, 1, 0, \dots)$, which in turn leads to the following outcome for effective labor inputs: $\{A_t\}_{t \in N} = (1, 2l, 1, 2l, 1, \dots)$.*

PROOF: *Delegated to the appendix.*

The intuition for this result is clear: Without a pension transfer the only remaining incentive for insiders is to raise their wages as much as possible without being fired. Recall that after such an episode there are no insiders anymore and markets clear again, and so forth. However, in the game theoretic sense as defined above this unique *outcome* may be produced by different *equilibria*; for example consider the following two different strategy profiles: (1) For all t the first one prescribes $q_t = 0$ for a history with $q_{t-1} = 1$, but $q_t = 1$ if $q_{t-1} \neq 1$. (2) For all t the second one simply mandates $q_t = 0$ for all $q_{t-1} \neq 0$ (for $q_{t-1} = 0$ the only feasible way is $q_t = 1$, see above). It is obvious that both strategy profiles contain mutually best responses of the players to other players' strategies, and that they both lead to the stated equilibrium outcome, because in this case the differences between the strategies are irrelevant, as they only relate to $q_{t-1} \neq 1$.

3.3 The full employment equilibria

The most interesting question in the context of the present model is whether full employment can be sustained as one equilibrium outcome. In this section I will present conditions on the production function f where this is indeed the case. In general there may be other equilibrium outcomes under these conditions with different strategy profiles (for example irregular outcomes), but see section 4 for important cases when this cannot happen. Note also that even the full employment outcome can result from several strategy profiles that will only differ with respect to their out-of-equilibrium strategies and thus there will be multiple full employment equilibria in the sense of definition 2. However, here I will focus on the simplest strategy profile, namely the one that prescribes to choose $q_t = 1$ for all insider unions $i = t \in N$, independently of the possible histories $h_t \in H_t$. For this rule I reach the following conclusion:

Lemma 2 *The strategy profile s^* which for all players $i = t \in N$ prescribes $q_t = 1$ for all possible histories $h_t \in H_t$ is a Nash equilibrium of the game $g(f; b, r, l)$ if (and only if)*

$$(11) \quad 1 = \arg \max_{q_t \in [0,1]} (1-b)f'[lq_t + 1] + \frac{b}{1+r}f'[\alpha]\alpha,$$

where $\alpha \equiv l(2 - q_t) + q_t = 2l + (1 - l)q_t$.

PROOF: *Delegated to the appendix.*

Thus whether the simple strategy profile $s^* : q_{t \in N} = 1$ is an equilibrium depends on the combination of the parameters b, r, l as well as on the production function. Since the direct own-wage effect of the first term is negative ($\partial f'[lq_t + 1]/\partial q_t < 0$ for all q_t), the indirect pension transfer effect of the second term must compensate, so it is necessary that the derivative $\partial(f'[\alpha]\alpha)/\partial q_t$ and by implication (the absolute value of) the elasticity of labor demand must be large enough, i.e. the inverse elasticity

$$(12) \quad \varepsilon[\alpha] \equiv \frac{f''[\alpha]\alpha}{f'[\alpha]} \leq 0$$

must be close enough to zero.⁵ To gain some intuition for the influence of the parameters, note that for $\varepsilon > -1$ lowering wages leads to a higher-than-proportional

⁵That ε is close enough to zero must hold at least for $q_t = q_{t-1} = q_{t+1} = 1$, i.e. for $\alpha = 1 + l$. Obviously the condition must also hold for other values of q_t , although strictly speaking there may be parameter regions where locally the indirect pension transfer effect does not dominate the direct own-wage effect. An important special case is obviously the situation where the former dominates the latter for all q_t , see the sufficient condition in proposition 1 below.

increase of employed labor and thus to a larger wage bill in the economy, and therefore $f'[\alpha]\alpha$ will rise with q_t . If that is the case, then it is clear that a higher contribution rate b and a lower discount factor r favor the indirect pension transfer effect. The same holds for lower unskilled labor productivity l , because a low l dampens the adverse effect of a rising q_t in the own-wage effect $f'[lq_t + 1]$ and at the same time amplifies the rise of the future wage bill to be taxed for pension benefits (i.e. yields a larger impact of q_t in $\alpha = 2l + (1 - l)q_t$). In that sense a higher b , a lower r , and a lower l make the outcome of full employment more likely in this model. In the spirit of this discussion it seems that condition (11) is perhaps too abstract, and to aid the interpretation I will now provide a set of sufficient and thus more restrictive conditions for which the given strategy profile is a Nash equilibrium.

Proposition 1 *If the following three conditions hold for all q_t they ensure that the strategy profile $s^* : q_{t \in N} = 1$ is a Nash equilibrium (i.e. they are jointly sufficient):*

$$(13) \quad \varepsilon[\alpha] > -1$$

$$(14) \quad \frac{\varepsilon[\alpha]}{1 + \varepsilon[\alpha]} \geq - \left(\frac{b}{1 - b} \right) \left(\frac{2}{1 + r} \right) (1 - l)$$

$$(15) \quad (f''' \geq 0 \ \& \ l \leq 0.5) \text{ or } (f''' \leq 0 \ \& \ l \geq 0.5)$$

PROOF: *Delegated to the appendix.*

The two inequalities (13) and (14) are the important and intuitive conditions specifying a lower bound for ε and thus a lower bound for the absolute value of the elasticity of labor demand. The previous discussion of the comparative statics of the parameters of course applies here as well. Condition (15) is technically needed to ensure sufficiency of the given condition set. Note that in the special cases $f''' = 0$ (or $l = 0.5$) sufficiency is ensured for any l (f''').

A simple example for a production function where the full employment equilibrium may exist is $f[x] = \sqrt{x}$ with a constant elasticity $\varepsilon[x] = -0.5$. For the sufficient conditions to hold we would need $l < 0.5$ since here $f'''[x] = (3/8)x^{-2.5} > 0$. Another example is $f[x] = x^3/6 - 0.5(1.01 + l)x^2 + (0.5(1 + l)^2 + 10)x$ with non-monotonous elasticity, but also with a positive third derivative: $f'''[x] = 1$.⁶

⁶The parameter l appears in the function to ensure that all required properties are met over the relevant range $x \in [0, 1 + l]$. The elasticity is given by $\varepsilon[x] = (x^2 - (1.01 + l)x) / (0.5x^2 - (1.01 + l)x + 0.5(1 + l)^2 + 10)$, which is decreasing for $x \approx 0$ but increasing for $x \approx 1 + l$.

3.4 Nash equilibria with partial outsider unemployment

This section analyzes equilibrium outcomes where some outsiders remain unemployed, but wages are lowered at least somewhat so that some outsiders are hired. Since for irregular equilibrium outcomes the players would face a serious coordination problem, steady-state equilibria with a clear focal point seem more attractive. Also, it is quite difficult to provide general conditions without a fully specified production function, and for these reasons I focus on steady-state outcomes. Of course, a steady state with $q_t = 0$ is impossible, because as described before, there will then be no insiders in the next period and the labor market will clear with $q_{t+1} = 1$. Bearing in mind again that profiles which only differ in their out-of-equilibrium strategies will lead to the same result, I will focus on the simplest possible strategy profile which prescribes to play $q_t = \bar{q}$ for all histories.

Lemma 3 *The strategy profile s^* which for all players $i = t \in N$ prescribes $q_t = \bar{q} \in (0, 1]$ for all possible histories $h_t \in H_t$ is a Nash equilibrium of the game $g(f; b, r, l)$ if*

$$(16) \quad \frac{\varepsilon[\beta]}{1 + \varepsilon[\beta]} = - \left(\frac{b}{1 - b} \right) \left(\frac{1 - l\bar{q}}{1 + r} \right) \bar{q} \left(1 + \frac{1}{l(2 - \bar{q})} \right),$$

with $\beta \equiv l\bar{q}(2 - \bar{q}) + \bar{q}$ (note that $\varepsilon > -1$ again is necessary), and if in addition:

$$(17) \quad f''[\beta](2 - \bar{q}) = f''[1 + l\bar{q}]$$

$$(18) \quad f''' \leq 0$$

PROOF: *Delegated to the appendix.*

Note that condition (18) is not strictly necessary for the entire range of the argument, but it ensures that the utility is globally maximized. Also, condition (17) implies that $f''' < 0$ must hold at least for some intervals, so the given sufficient condition set is not very different from the necessary set. Conditions (16) and (17) together are very restrictive but are actually necessary; the reason is that the same employment quota choice must be optimal after the initial condition $q_0 = 1$ as well as in the steady state $q_{t-1>0} = \bar{q}$. Thus this result with interior solutions is not as attractive as the full employment corner solution of the previous section. An example of the narrow class of production functions for which this result applies is given in the proof of lemma 3 in the appendix.

4 Forward induction solutions

In this section I will investigate the stronger solution concept of forward induction that can be applied to the model. The economy is still characterized by the game structure $g(f; b, r, l)$ although no game theoretic equilibrium concepts will be needed here. The following definition applies:

DEFINITION 3 *The sequence $\{q_t^{fw}\}_{t \in N}$ is a forward induction equilibrium of the economy $g(f; b, r, l)$ if (and only if) for all $t \in N$ and $q_{t+1} \in [0, 1]$ we have*

$$(19) \quad q_t^{fw} = \arg \max_{q_t \in [0,1]} V_{i=t}(q_{t-1}^{fw}, q_{t+1}; q_t),$$

where as before the initial condition $q_0^{fw} = 1$ holds.

As the name suggests these equilibria –if they exist– can be found by iteratively solving the utility maximization problem for each period starting at the initial condition $q_0^{fw} = 1$. However, we can only avoid the problem of forming expectations about the respective future q_{t+1} if there is a unique maximizer q_t^{fw} for all possible values of q_{t+1} . Therefore this solution concept is quite a bit stronger than a Nash equilibrium. The advantages are that we only need the assumption of individual utility maximization and that there can only be one unique equilibrium outcome. I will address the same outcomes that I analyzed with game theory, namely the case of no pension system, the full employment equilibrium, and steady-state equilibria with partial outsider unemployment. It turns out that the latter is impossible, and in addition I will show that irregular equilibria with partial outsider unemployment are also impossible.

Let us first briefly look at the case of no pension system again ($b = 0$), which produces a rather obvious result:

Lemma 4 *If $b = 0$, the Nash equilibrium outcome is identical to the forward induction solution.*

PROOF: *Delegated to the appendix.*

A more interesting question concerns the condition under which full employment could be inferred without resorting to game theory. To this end I first apply the above definition to our concrete model structure to obtain:

Lemma 5 *If (and only if) for all $q_{t+1} \in [0, 1]$ it holds that*

$$(20) \quad 1 = \arg \max_{q_t \in [0,1]} (1-b)f'[lq_t + 1] + \frac{b}{1+r} f'[lq_{t+1}(2-q_t) + q_t] (lq_{t+1}(2-q_t) + q_t),$$

then the full employment path $\{q_t = 1\}_{t \in N}$ is also the forward induction solution.

PROOF: *Delegated to the appendix.*

As a second step, since for a given f it will often be difficult to establish the abstract condition (20) directly, the following result is useful, which provides some relatively straightforward sufficient conditions to check whether a specific economy $g(f; b, r, l)$ will display full employment as the unique forward induction solution:

Proposition 2 *If an economy $g(f; b, r, l)$ satisfies the sufficient conditions for the full employment Nash equilibrium given in proposition 1, and if in addition the production function is such that ε is a non-increasing function, then the full employment outcome is the unique equilibrium and could also have been derived by forward induction.*

PROOF: *Delegated to the appendix.*

As an example consider again $f[x] = \sqrt{x}$ with $\varepsilon[x] = -0.5$, so if parameter values satisfy the sufficient conditions for a full employment Nash equilibrium, this would be the forward induction equilibrium as well. Note that the special case $f''' \leq 0$ implies $d\varepsilon/dx < 0$, so only functions with $f''' > 0$ may lead to full employment Nash equilibria that are not forward induction equilibria. To sum up what we can learn from the sufficient conditions with respect to the existence of full employment equilibria figure 3 presents the various criteria in a form similar to a decision tree. The term ‘‘Nash’’ indicates that the forward induction sufficient condition does not apply, but there also may or may not be such an equilibrium. Recall that on the other hand the forward induction equilibrium implies the Nash equilibrium.

Finally it remains to be checked if there are conditions which lead to a forward induction equilibrium that is not a corner solution, i.e. such that for all t we have $q_t^{fw} \in (0, 1)$. Already by intuition it is clear that because q_t^{fw} must be optimal for any possible q_{t+1} and also both after the initial condition $q_0 = 1$ and afterwards for $q_{t-1} < 1$, such a solution is very unlikely to exist. The following proposition confirms and strengthens this intuition.

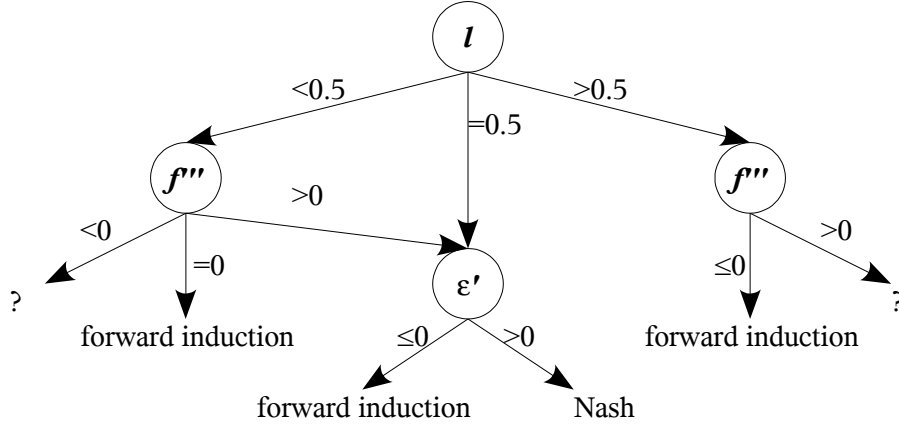
Proposition 3 *A forward induction equilibrium with partial outsider unemployment does not exist.*

PROOF: *Delegated to the appendix.*

Note that the proposition covers steady-state as well as irregular equilibria. Thus for $b \neq 0$ the economy either has full employment as the forward induction equilibrium, or no forward induction solution at all. (But of course it may have Nash equilibria in the latter case.)

Finally, let me briefly explain why the analysis of the standard Nash equilibrium refinement concept of subgame perfection was omitted in this paper. In principle, subgame perfection is desirable because it explicitly takes into account the

Figure 3: Full employment equilibrium implications of the technical conditions



Notes: This tree is applicable if condition (14) holds. It shows which full employment equilibrium is known to exist based on the sufficient condition (15) and proposition 2. A question mark indicates that neither sufficient condition applies, and there may or may not be an equilibrium.

sequential nature of extensive games and requires the strategies of the players to be optimal after any conceivable history, and not only along the equilibrium path. In the present game, the relevant history for any period t is just q_{t-1} , so subgame perfection would formally require that $V_{i=t}(q_{t-1}, q_{t+1} = 1; 1) \geq V_{i=t}(q_{t-1}, q_{t+1} = 1; q_t)$ for all q_{t-1} and q_t .⁷ However, it turns out that in our case subgame perfection of the full employment equilibrium requires $f''' < 0$.⁸ But $f''' < 0$ implies that ε is a decreasing function, such that we know from proposition 2 that a Nash equilibrium that satisfies the conditions in proposition 1 automatically produces the unique forward induction solution, which must correspond to the outcome of some subgame perfect equilibrium. Hence little would have been gained by separately analyzing subgame perfect equilibria.

⁷In terms of the formal game description, q_{t-1} here is sufficient to capture all possible histories $h_t \in H_t$, and $q_{t+1} = 1$ represents the relevant equilibrium strategies of future players ($s_{>i}^*$). The equilibrium strategies of past players ($s_{<i}^*$, determining q_{t-1} in equilibrium) are irrelevant here, since for an analysis of subgame perfection we have to check the condition for all possible histories q_{t-1} anyway.

⁸More precisely, the sufficient condition set that I was able to find includes the restriction $f''' < 0$; see the appendix for the proof.

5 *Conclusions and discussion*

This paper presented a highly stylized OLG model combining insider-outsider unemployment theory with a generational conflict due to the earlier arrival of older workers on the labor market, which gives them insider power. It was also assumed that workers' productivity is enhanced through learning by doing. Then it could be shown that a pay-as-you-go pension system can induce insiders to prevent outsider unemployment, because the pension system lets them participate in the increased earnings potential of (former) outsiders. However, this effect outweighs the necessary reduction of wages only if labor demand is sufficiently elastic.

At first sight it may seem possible to apply the argument of this model to an individually funded pension system as well: Higher employment levels could raise the marginal productivity of capital and therefore lead to higher interest rates, which could provide the necessary incentive for insiders who are accumulating funds for their retirement. A closely related and rigorous version of this argument can be found in WIGGER [2001]. However, the major caveat is that this effect would only work in a closed economy, because long-run real interest rates of an open economy are mainly determined in the world capital market rather than at home.⁹

It is clear that the central argument in this paper should not be interpreted as the simplistic belief that a pay-as-you-go pension scheme is good and funded systems are bad. For instance, demographic changes were assumed away, including those that are presently putting pressure on unfunded pension schemes in industrial countries. I do not claim, either, that the effect described in this paper accounts for the virtual omnipresence of pay-as-you-go pension systems in insider-plagued economies. Apart from the case of dynamically inefficient economies, these institutions with intergenerational transfers are typically installed in times of imminent urgency, i.e. when older generations face poverty because the economy's capital stock has been wiped out, be it because of wars or after natural catastrophes. This distributional aspect of pension systems is clearly more important than any of the efficiency reasons one can find. However, there is an ongoing discussion about the future of unfunded pensions in industrial countries, and therefore it is useful to shed some light on the effect that pension systems can have in mature economies with otherwise persistent insider unemployment problems. Also, note that the

⁹The analysis could also relate to a smaller economic entity. The owner of a small firm could be induced to hire somebody who could become his successor, even if the worker were not profitable initially. The acquisition of firm-specific human capital on the part of the worker would increase his reservation utility of taking over the firm. Hence the owner could charge a higher price, rewarding him for the earlier implicit employment "subsidy". (This interpretation was pointed out by Hans Friederiszick.)

phenomenon called “unemployment” in the present paper need not imply that a part of the labor force is completely unused. Instead, adopting a dual labor market framework, “employment” could be interpreted as employment in the “good jobs” sector, whereas “unemployed” workers would have to conduct tasks in the “bad jobs” sector, where they cannot acquire any skills.

It should be noted that an essential assumption is that the decisions of centralized (or coordinated) insider unions affect the entire labor force. Thus I described effects arising in a corporatist economy, otherwise the argument would be weakened by free-rider incentives. But other interpretations of the general setup are also possible: In this model insiders were rewarded for not causing outsider unemployment through higher pensions, but other elements of the welfare state that are financed by insiders’ taxes could in principle serve a similar purpose. However, whereas pension systems usually have their own budget, it is probably more difficult to link lower unemployment to lower taxes at some point in time. In general, the whole issue is related to the popular argument by CALMFORS AND DRIFFILL [1988], claiming that centralized institutions take their possibly adverse (side) effects better into account than less coordinated institutional arrangements.

Appendix: Proofs

PROOF of lemma 1: With $b = 0$ the utility of insiders reduces to $V_{i=t} = f'[lq_t(2 - q_{t-1}) + q_{t-1}]$, and the choice variable q_t enters positively into a decreasing function. Therefore it is optimal to choose $q_t = 0$, and this determines the equilibrium outcome for every second period with existing insider unions. Recall that after a period with $q_t = 0$ there are no insiders and the economy is restricted to clearing markets, i.e. $q_{t+1} = 1$. This completes the cycle, and the cycle for A_t follows directly. *Q.E.D.*

PROOF of lemma 2: The stated condition is the direct application of the definition of Nash equilibrium. For the analyzed strategy profile the given strategies of the other players $s_{<i}^*$ and $s_{>i}^*$ lead to $q_{t-1} = q_{t+1} = 1$, and therefore it has to be the case that $q_t = 1$ maximizes $V_{i=t}(1, 1; q_t)$ which is just the right-hand side of equation (11). This also encompasses the first player’s situation because for $i = t = 1$ we have $q_0 = 1$ by construction, as discussed before. *Q.E.D.*

PROOF of proposition 1: The idea behind the sufficient condition set is to ensure that insiders’ utility for given $q_{t-1} = q_{t+1} = 1$, i.e. $V_{i=t}(1, 1; q_t)$, is increasing for all q_t which makes the corner solution $q_t = 1$ the optimal choice. (It

is of course not *necessary* for the existence of a full employment Nash equilibrium that $V_{i=t}(1, 1; q_t)$ is monotonous.) An equivalent problem is to ensure that $(1/f'[\alpha])\partial V_{i=t}(1, 1; q_t)/\partial q_t \geq 0$ for all q_t , which is given by:

$$(A1) \quad \frac{1}{f'[\alpha]} \frac{\partial V_{i=t}(1, 1; q_t)}{\partial q_t} = (1-b) \frac{f''[lq_t+1]}{f'[\alpha]} l + \frac{b}{1+r} (1-l) (1 + \varepsilon[\alpha])$$

Since the first summand is negative, $\varepsilon[\alpha] > -1$ is necessary to make the entire expression positive. Furthermore we can write

$$(A2) \quad \frac{f''[lq_t+1]}{f'[\alpha]} = \frac{f''[\alpha]}{f'[\alpha]} + \Delta,$$

where $\Delta \equiv (f''[lq_t+1] - f''[\alpha])/f'[\alpha]$, so I can now write the restriction on the derivative of insiders' utility as:

$$(A3) \quad (1-b) \left(\frac{\varepsilon[\alpha]}{\alpha} + \Delta \right) l + \frac{b}{1+r} (1-l) (1 + \varepsilon[\alpha]) \geq 0$$

A few rearrangements lead to:

$$(A4) \quad \frac{\varepsilon[\alpha] + \Delta\alpha}{1 + \varepsilon[\alpha]} \geq - \left(\frac{b}{1-b} \right) \left(\frac{1}{1+r} \right) \frac{(1-l)^2}{l} \left(\frac{2l}{1-l} + q_t \right)$$

To arrive at the stated conditions, note the following properties: First, the critical value for $(\varepsilon[\alpha] + \Delta\alpha)/(1 + \varepsilon[\alpha])$ depends on q_t . This is admissible but inconvenient. It can easily be seen that the maximum of the critical value is attained for $q_t = 0$, wherefore I choose that value for the sufficient condition. Second, if $\Delta \geq 0$ then $\Delta\alpha \geq 0$ (remember $\alpha \equiv l(2 - q_t) + q_t$), and in that case it will be sufficient if $\varepsilon[\alpha]/(1 + \varepsilon[\alpha])$ exceeds the critical value, since the left-hand side of (A4) will be even larger. Thus it is useful to determine in which cases $\Delta \geq 0$ holds. Assuming a monotonous function f'' , a comparison of the arguments of the functions in the numerator of Δ makes it clear that the stated condition (15) names all possibilities for this, because $lq_t + 1 \geq \alpha \Leftrightarrow 0 \geq (2l - 1)(1 - q_t)$. This implies that conditions (13), (14), and (15) are sufficient for (A4) to hold, which ensures optimality of $q_t = 1$. *Q.E.D.*

PROOF of lemma 3: Using the given strategies of the stated strategy profile the utility of all insiders except in the first period is given by $V_{i=t>1}(\bar{q}, \bar{q}; q_t)$. In the first period insiders face the initial condition $q_0 = 1$, and their utility is thus $V_1(1, \bar{q}; q_t)$. Hence both these utilities must be maximized for the choice $q_t = \bar{q}$ to ensure this steady state equilibrium.

I start with the first-order condition for the bulk of insiders:

$$(A5) \quad \frac{\partial V_{i=t>1}(\bar{q}, \bar{q}; q_t)}{\partial q_t} = (1-b)f''[lq_t(2-\bar{q}) + \bar{q}]l(2-\bar{q}) + \frac{b}{1+r} * \\ (f''[l\bar{q}(2-q_t) + q_t](1-l\bar{q})(l\bar{q}(2-q_t) + q_t) + f'[l\bar{q}(2-q_t) + q_t](1-l\bar{q})) \\ = \Big|_{q_t=\bar{q}} 0$$

Replacing q_t with \bar{q} , using the definition $\beta \equiv l\bar{q}(2-\bar{q}) + \bar{q}$, and dividing by $f'[\beta]$ yields the following relation:

$$(A6) \quad (1-b)\frac{\varepsilon[\beta]}{\beta}l(2-\bar{q}) + \frac{b}{1+r}(1-l\bar{q})(1+\varepsilon[\beta]) = 0$$

Again it is clear that $1 + \varepsilon[\beta] > 0$ must hold, and thus dividing by $(1 + \varepsilon[\beta])(1 - b)l(2 - \bar{q})/\beta$ and re-substituting β leads to (16).

For the first-period insiders the first-order condition is a little different:

$$(A7) \quad \frac{\partial V_{i=t=1}(1, \bar{q}; q_t)}{\partial q_t} = (1-b)f''[lq_t + 1]l + \frac{b}{1+r} * \\ (f''[l\bar{q}(2-q_t) + q_t](1-l\bar{q})(l\bar{q}(2-q_t) + q_t) + f'[l\bar{q}(2-q_t) + q_t](1-l\bar{q})) \\ = \Big|_{q_t=\bar{q}} 0$$

Since the second summand is unchanged it is clear that the first one must also be equal for $q_t = \bar{q}$: $f''[\beta](2-\bar{q}) = f''[l\bar{q} + 1]$. Note that the inequality $\beta = l\bar{q}(2-\bar{q}) + \bar{q} = 2l\bar{q} - l\bar{q}^2 + \bar{q} < l\bar{q} + 1$ becomes $\bar{q}(1-l\bar{q}) < 1-l\bar{q}$ after subtracting $2l\bar{q}$ and thus is true. It follows that f''' must at least “on average” be negative between β and $l\bar{q} + 1$. I impose the slightly stronger condition that $f''' < 0$ everywhere, also with a view to the second-order condition, where for brevity I suppress the arguments of the various derivatives:

$$(A8) \quad \frac{\partial^2 V_{i=t>1}(\bar{q}, \bar{q}; q_t)}{\partial q_t^2} = \\ (1-b)l^2(2-\bar{q})^2 f''' + \frac{b}{1+r}(1-l\bar{q})^2 (f''' * (l\bar{q}(2-q_t) + q_t) + 2f'')$$

It is clear that $f''' < 0$ is a sufficient condition to ensure that this expression is negative. For $f''' < 0$ the second-order condition also shows that $V_{i=t>1}(\bar{q}, \bar{q}; q_t)$ is strictly concave in q_t , and thus an interior maximum will also be a global one. The second-order condition for the first-generation insiders is as follows:

$$(A9) \quad \frac{\partial^2 V_{i=t=1}(1, \bar{q}; q_t)}{\partial q_t^2} = \\ (1-b)l^2 f''' + \frac{b}{1+r}(1-l\bar{q})^2 (f''' * (l\bar{q}(2-q_t) + q_t) + 2f'')$$

It can be seen that $f''' < 0$ also ensures concavity of $V_{i=t=1}(1, \bar{q}; q_t)$.

An example can be constructed from (17) by treating the right-hand side $f''[1 + l\bar{q}]$ as a constant parameter, letting q_{t-1} vary in the left-hand side $f''[2l\bar{q} + (1 - l\bar{q})q_{t-1}](2 - q_{t-1})$, and integrating twice. The result is

$$(A10) \quad f[x] = -k_3(l\bar{q} - 1)^2 \left(2 + \frac{2l\bar{q}}{1 - l\bar{q}} \right) \left(\ln \left(2 + \frac{2l\bar{q}}{1 - l\bar{q}} \right) - 1 \right) + \\ k_2x + k_3(l\bar{q} - 1)^2 \left(2 - \frac{x - 2l\bar{q}}{1 - l\bar{q}} \right) \left(\ln \left(2 - \frac{x - 2l\bar{q}}{1 - l\bar{q}} \right) - 1 \right),$$

which satisfies $f, f' > 0$ for sufficiently large values of $k_2 > 0$ as well as $f[0] = 0$ and $f'', f''' < 0$ for $k_3 < 0$; the parameter k_3 is just another name for $f''[1 + l\bar{q}]$. The function is written in such a way that for given economy parameters b, r, l and a choice of k_2, k_3 the equilibrium value \bar{q} (if it exists) can be found by solving condition (16) with numerical methods. For example, setting $k_2 = 25, k_3 = -10, l = 0.5, b = 0.2, r = 0.3$ (remember that the discount period is an entire generation) yields an equilibrium value of $\bar{q} \approx 0.18$. Having \bar{q} (and l) in the function definition is necessary to restrict its shape such that an equilibrium is possible, but this also makes it obvious that the conditions are extremely demanding. *Q.E.D.*

PROOF of lemma 4: Since the value of q_{t+1} is irrelevant in the proof of lemma 1 and the utility function is the same, that proof directly applies here as well.

Q.E.D.

PROOF of lemma 5: Note that condition (20) uses the insider union's utility for given $q_{t-1} = 1$, i.e. $V_{i=t}(1, q_{t+1}; q_t)$. First consider the starting period $t = 1$, where $q_0 = 1$ by construction as before. Under condition (20) the insider union in period $t = 1$ will therefore set $q_1 = 1$, no matter what it expects for q_2 (or if it forms any expectation at all). Hence the insider union of the next period $t = 2$ faces exactly the same problem and therefore also chooses $q_2 = 1$, and so forth for all $t \in N$. For this proof no game theory was used. *Q.E.D.*

PROOF of proposition 2: To see that $d\varepsilon[x]/dx \leq 0$ together with the conditions of proposition 1 is sufficient for the forward induction condition (20) note the following: The sufficient conditions in proposition 1 guarantee that $\partial V_{i=t}(1, 1; q_t)/\partial q_t \geq$

0 for all q_t which makes $q_t = 1$ optimal. If for all q_t it can be shown that for any given $q_{t+1} < 1$ the slope of the utility function is even bigger,

$$\partial V_{i=t}(1, q_{t+1}; q_t) / \partial q_t > \partial V_{i=t}(1, 1; q_t) / \partial q_t,$$

it would strengthen the optimality of the corner solution q_t . A sufficient condition for this is that the slope of the utility is decreasing in q_{t+1} , and therefore I have to check whether $\partial^2 V_{i=t}(1, q_{t+1}; q_t) / \partial q_t \partial q_{t+1} < 0$ for all q_t . This is given by

$$(A11) \quad \frac{\partial^2 V_{i=t}(1, q_{t+1}; q_t)}{\partial q_t \partial q_{t+1}} = \frac{b}{1+r} * \left(-l(1+\varepsilon)f' + (1-lq_{t+1}) \left(\frac{d\varepsilon}{dx}(2-q_t)lf' + (1+\varepsilon)f''(2-q_t)l \right) \right),$$

where the function argument $lq_{t+1}(2-q_t) + q_t$ has been suppressed to improve readability. Given that $f' > 0$, $f'' < 0$ and $1+\varepsilon > 0$ is necessary for a Nash equilibrium, it is easy to verify that $d\varepsilon/dx \leq 0$ (over the relevant range of $x \in [0, 1+l]$) is sufficient for the entire cross-derivative to be negative. *Q.E.D.*

PROOF of proposition 3: For ease of exposition the proof proceeds in two steps: First I show that a steady-state forward induction equilibrium with partial outsider unemployment is impossible, then I extend the analysis to the general interior case.

The steady-state step: Recall that to infer the path $\{\bar{q}\}_{t \in N}$ by forward induction, it must hold that \bar{q} is the maximizer of the utility $V_{i=t}(q_{t-1}, q_{t+1}; q_t)$ for all possible values $q_{t+1} \in [0, 1]$ as well as for the two possible values $q_{t-1} \in \{\bar{q}, 1\}$. Let us first suppose that for some pair $(q_{t-1}^* = \bar{q}, q_{t+1}^* \neq \bar{q})$ it is indeed optimal to choose $q_t = \bar{q}$, which above all means that the relevant first-order condition holds: $\partial V_{i=t}(\bar{q}, q_{t+1}^*; q_t) / \partial q_t = |_{q_t=\bar{q}} 0$. Then I would need to show that this condition still holds for varying q_{t+1} . Since q_{t+1} does not affect the first summand of the derivative of (9) (w.r.t. q_t), the remaining term must not change, either: $\partial^2 \gamma / (\partial q_t \partial q_{t+1}) = |_{q_t=\bar{q}} 0$, where the second summand of (9) is abbreviated by $\gamma \equiv b/(1+r)(f'lq_{t+1}(2-q_t) + q_t)$. Writing out this condition in full and dividing by $b/(1+r)$ yields the following equation, where again the function argument $lq_{t+1}(2-\bar{q}) + \bar{q}$ is suppressed for readability:

$$(A12) \quad (1-lq_{t+1})^2(f''' * (lq_{t+1}(2-\bar{q}) + \bar{q}) + 2f'') - l(f'' * (lq_{t+1}(2-\bar{q}) + \bar{q}) + f') = 0$$

Dividing by f' yields:

$$\Leftrightarrow (1 - l_{q_{t+1}})^2 \left(\frac{\varepsilon f'''}{f''} + \frac{2\varepsilon}{l_{q_{t+1}}(2 - \bar{q}) + \bar{q}} \right) - l(1 + \varepsilon) = 0$$

$$(A13) \quad \Leftrightarrow \frac{\varepsilon}{1 + \varepsilon} \left(\frac{f'''}{f''} + \frac{2}{l_{q_{t+1}}(2 - \bar{q}) + \bar{q}} \right) (1 - l_{q_{t+1}})^2 = l$$

But a necessary condition for this is that the term in big parentheses is negative, since $\varepsilon/(1 + \varepsilon)$ must already be negative. Therefore it must be true that

$$(A14) \quad f'''[l_{q_{t+1}}(2 - \bar{q}) + \bar{q}] * (l_{q_{t+1}}(2 - \bar{q}) + \bar{q}) > -2f''[l_{q_{t+1}}(2 - \bar{q}) + \bar{q}] > 0$$

for all possible values of q_{t+1} . However, picking $q_{t+1} = \bar{q}$ and comparing this to the second-order condition (A8) reveals that this implies

$$\partial^2 V_{i=t>1}(\bar{q}, \bar{q}; q_t) / \partial q_t^2 >|_{q_t=\bar{q}} 0.$$

Hence the choice $q_t = \bar{q}$ cannot be optimal for $q_{t+1} = \bar{q}$, given optimality for any other $q_{t+1} \neq \bar{q}$ (and the history $q_{t-1} = \bar{q}$), and a forward induction equilibrium with constant partial unemployment is thus impossible.

The second step covering irregular equilibria as well: Here I start from the initial condition $q_0 = 1$. Suppose that for insiders in $t = 1$ it were indeed optimal to choose $q_1^* < 1$ for any possible value q_2 , given the history $q_0 = 1$. As should be clear from the first step of this proof, apart from the first-order condition $\partial V_1(1, q_2; q_1) / \partial q_1 =|_{q_1=q_1^*} 0$ we would have the analogue to the necessary forward-induction condition (A14):

$$(A15) \quad f'''[l_{q_2}(2 - q_1^*) + q_1^*] * (l_{q_2}(2 - q_1^*) + q_1^*) > -2f''[l_{q_2}(2 - q_1^*) + q_1^*] > 0$$

for all values of q_2 . Next suppose that also for insiders in $t = 2$ there is some (possibly different) optimal choice q_2^* for any possible value of q_3 given the history q_1^* . Note that picking this value q_2^* for q_2 in (A15) implies $f'''[l_{q_2^*}(2 - q_1^*) + q_1^*] > 0$.

The second-period analogy to (A15) implies that $f'''[l_{q_3}(2 - q_2^*) + q_2^*] * (l_{q_3}(2 - q_2^*) + q_2^*) + 2f''[l_{q_3}(2 - q_2^*) + q_2^*] > 0$ must hold for all values of q_3 . Now let us check the second-order condition for optimality in $t = 2$:

$$(A16) \quad \frac{\partial^2 V_2(q_1^*, q_3; q_2)}{\partial q_2^2} =|_{q_2=q_2^*} (1 - b)l^2 f'''[l_{q_2^*}(2 - q_1^*) + q_1^*] + \frac{b}{1 + r} * (1 - l_{q_3})^2 (f'''[l_{q_3}(2 - q_2^*) + q_2^*] (l_{q_3}(2 - q_2^*) + q_2^*) + 2f''[l_{q_3}(2 - q_2^*) + q_2^*])$$

It is now obvious from the previously derived necessary conditions that the expression in (A16) is positive rather than negative. Thus q_2^* cannot be optimal, in contradiction to the assumption, and I conclude that a forward induction equilibrium path with partial outsider unemployment is impossible. *Q.E.D.*

PROOF of the requirement mentioned in footnote 8: This proof is much like the one for the Nash equilibrium in proposition 1. A sufficient condition that the strategy profile of proposition 1 is also subgame perfect is to ensure that insiders' utility is monotonously increasing in q_t for all q_{t-1} , given $s_{>i}^* : q_{t+1} = 1$. Formally, $\partial V(q_{t-1}, 1; q_t)/\partial q_t > 0 \Leftrightarrow (1/f'[\alpha])\partial V(q_{t-1}, 1; q_t)/\partial q_t > 0$, and this latter inequality evaluates to:

$$(1-b) \frac{f''[lq_t(2-q_{t-1})+q_{t-1}]}{f'[\alpha]}(2-q_{t-1})l + \frac{b}{1+r}(1-l)(1+\varepsilon[\alpha]) > 0$$

$$(A17) \quad \Leftrightarrow (\varepsilon[\alpha] + \Delta_2\alpha)(2-q_{t-1}) > -\alpha \frac{b}{1-b} \frac{1-l}{l} \frac{1+\varepsilon[\alpha]}{1+r}$$

In analogy to the sufficient condition for the Nash equilibrium I have used a new $\Delta_2 \equiv (f''[lq_t(2-q_{t-1})+q_{t-1}] - f''[\alpha])/f'[\alpha]$ here, and again I restrict Δ_2 to be positive, this time for all q_{t-1} . Picking $q_{t-1} = 0$ then means that $f''[2lq_t] - f''[\alpha = 2l + (1-l)q_t] > 0$ should hold. Since $2lq_t < 2l + q_t(1-l)$, this implies that $f''' < 0$ (assuming a monotonous f'') is required as part of this sufficient condition for subgame perfection.

Q.E.D.

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Sven Schreiber

Faculty of Economics and Business, Department of Money & Macro

Goethe University Frankfurt

Mertonstr. 17, PF 82

D-60054 Frankfurt

Germany

E-mail:

sschreiber@wiwi.uni-frankfurt.de